Solve:

$$x^2 \frac{d^2y}{dx^2} + y = 3x^2 \longrightarrow (1)$$

Ans: Eq^n (1) is 2nd order linear homogeneous.

Putting $x = e^z$, the above equation reduces to

$$D(D-1)y + y = 3e^{2z}, \quad \frac{d}{dz} = D$$

$$\text{or, } (D^2 - D + 1)y = 0 \longrightarrow (2)$$

Eq^n (2) is 2nd order linear.

Let, $y = e^{mz}$ $(\neq 0)$ be a trial sol^n of eq^n (2)

$\therefore$ A.E is — $m^2 - m + 1 = 0$

or, $m = \frac{1 \pm \sqrt{3}i}{2}$

$$\therefore C.F = \left(c_1 \cos \tfrac{\sqrt{3}}{2} z + c_2 \sin \tfrac{\sqrt{3}}{2} z\right) e^{z/2}$$

$$P.I = \frac{1}{D^2 - D + 1} \cdot 3e^{2z}$$

$$= \frac{1}{3} \cdot 3e^{2z}$$

$$= e^{2z}$$

$$= x^2$$

$\therefore$ General sol^n $y = C.F + P.I$

$$= \left(c_1 \cos \frac{\sqrt{3}}{2} z + c_2 \sin \frac{\sqrt{3}}{2} z\right) e^{z/2} + x^2$$

2. $x^3 \frac{d^3y}{dx^3} + 2x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = x^2 \log x.$

Ans: Eq^n (1) is 3rd order linear homogeneous.

08am

∴ Putting $x = e^z$, the above eq^n reduced to

09am

$D(D-1)(D-2)y + 2D(D-1)y - Dy + y = e^{2z}.z$

10am

$(D^3 - 3D^2 + 2D)y + (2D^2 - 2D)y - Dy + y = 0$

11am

or, $(D^3 - 3D^2 + 2D + 2D^2 - 2D - D + 1)y = 0$

Noon

or, $(D^3 - D^2 - D + 1)y = 0. \longrightarrow (2)$

01pm

Let, $y = e^{mz}$ $(\neq 0)$ be a trial sol^n of Eq^n (2)

02pm

∴ Its A.E is $m^3 - m^2 - m + 1 = 0$

03pm

or, $m^2(m-1) - (m-1) = 0$

04pm

or, $(m-1)(m^2-1) = 0$

05pm

or, $(m-1)(m+1)(m-1) = 0$

Eve

or, $m = 1, 1, -1.$

07pm

∴ $C.F = (c_1 + c_2 z)e^z + c_3 e^{-z} = (c_1 + c_2 \log x)x + \frac{c_3}{x}$

Now, $P.I = \frac{1}{D^3 - D^2 - D + 1} . ze^{2z}$

Work to do

$= e^{2z} \frac{1}{(D+2)^3 - (D+2)^2 - (D+2) + 1} . z.$

$$= e^{2z} \cdot \frac{1}{D^3+6D^2+12D+8-D^2-4D-4-D-2+1} \cdot z$$

08am $$= e^{2z} \cdot \frac{1}{D^3+5D^2+7D+3} \cdot z$$

09am $$= e^{2z} \cdot \frac{1}{3\left(1+\frac{D^3+5D^2+7D}{3}\right)} \cdot z$$

10am

11am $$= \frac{e^{2z}}{3} \cdot \left(1+\frac{D^3+5D^2+7D}{3}\right)^{-1} \cdot z$$

Noon $$= \frac{e^{2z}}{3} \cdot \left(1-\frac{D^3+5D^2+7D}{3}+\cdots\right) \cdot z$$

01pm $$= \frac{e^{2z}}{3} \cdot \left(z-\frac{7}{3}\right)$$

02pm $$= \frac{x^2}{3} \cdot \left(\log x-\frac{7}{3}\right)$$

03pm $$= \frac{x^2}{9}(3\log x-7).$$

04pm $\therefore$ General solⁿ, $y = C.F + P.I$

05pm $$= (c_1+c_2\log x)x + \frac{c_3}{x} + \frac{x^2}{9}(3\log x-7)$$

Eve